

Grade 10 MYP
Unit 2 Geometry
C: Trigonometry

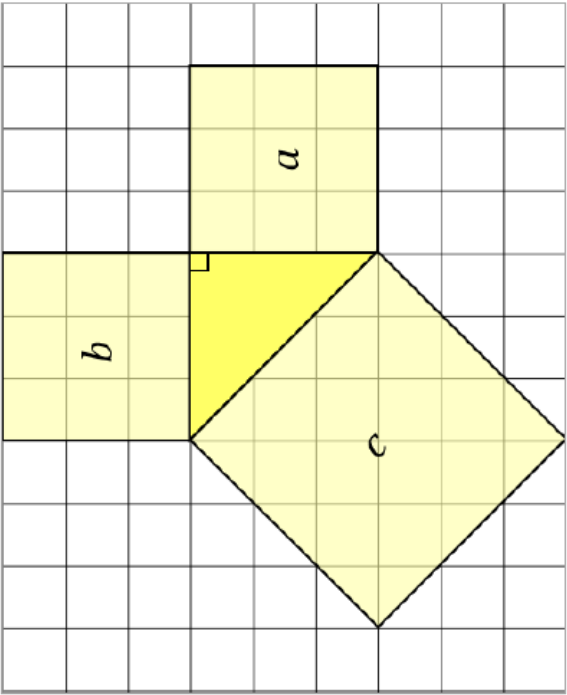
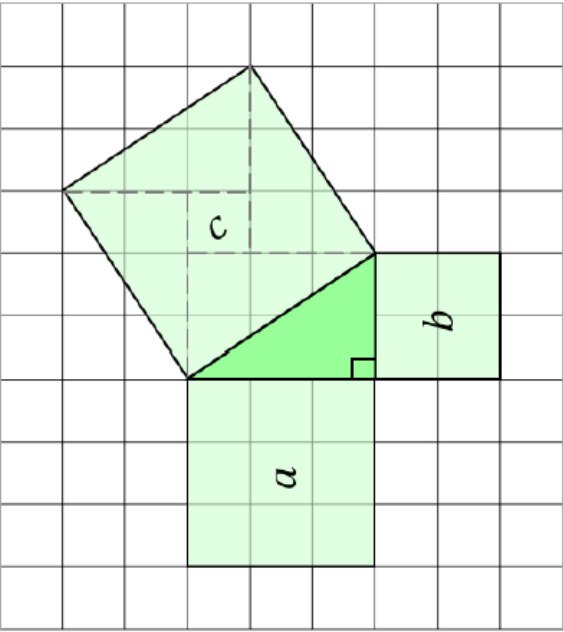
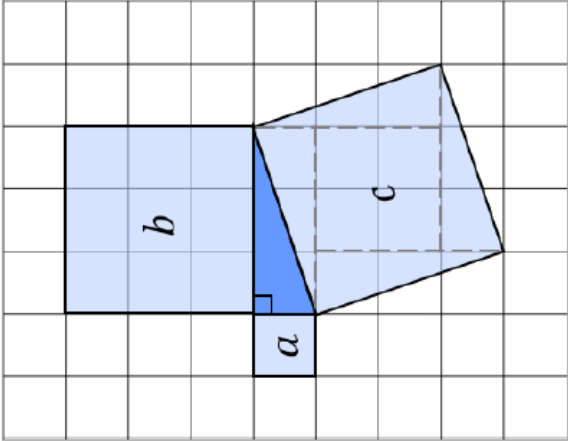
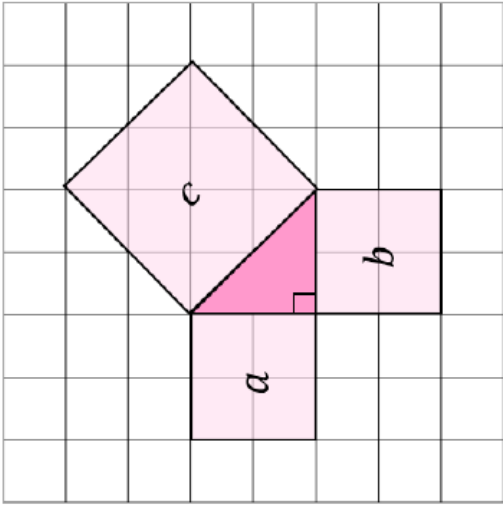
Name _____

C: Trigonometry	Pythagoras Theorem
	Review: SOHCAHTOA finding missing angles
	Review: SOHCAHTOA finding missing sides
	Angles of elevation, depression and bearings
	Sine rule
	Cosine rule
	Area of non right angled triangles
	Area of a segment

Write key facts here:

Investigating Right-Angled Triangles

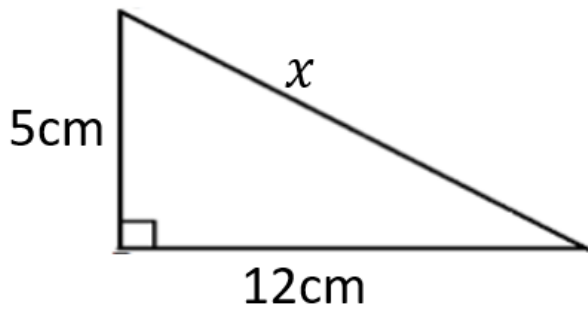
For each of the following right-angled triangles, find the areas of the squares attached to each of the sides.



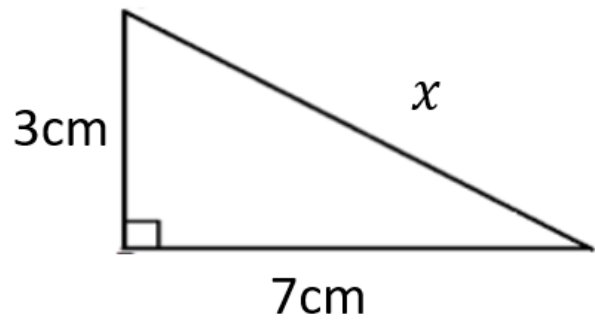
Triangle	Area a	Area b	Area c
Pink			
Blue			
Green			
Yellow			

What do you notice? _____

Worked Example



Your Turn

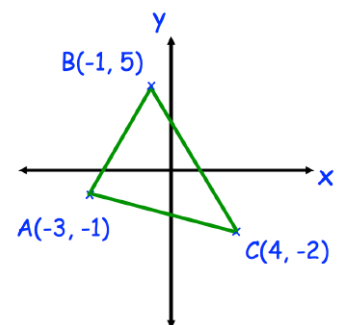


Test your understanding

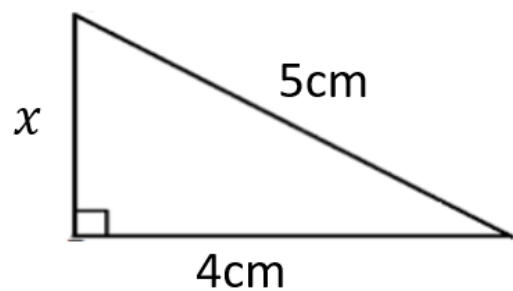
(g) Find x to 1 decimal place	(h) Find y	(i) Find y to 1 decimal place
(j) Find x , leaving your answer as a surd	(k) Find y , leaving your answer as a surd	(l) Find x , leaving your answer as a surd

Extension

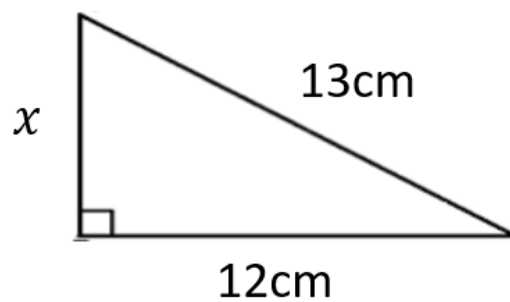
Question 1: Calculate the perimeter of triangle ABC.



Worked Example

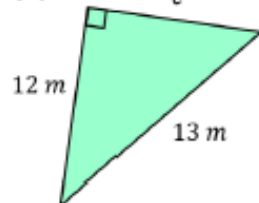


Your Turn

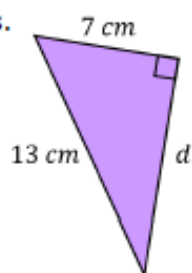


Find the missing lengths.

(a)

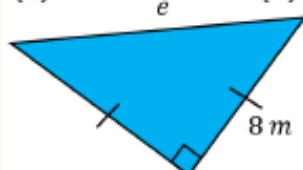


(b)

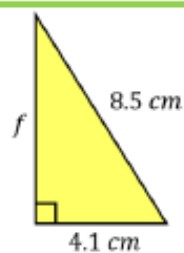


Find the missing lengths.

(a)



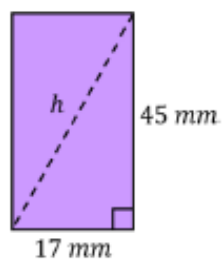
(b)



(c)

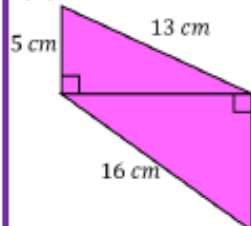


(d)

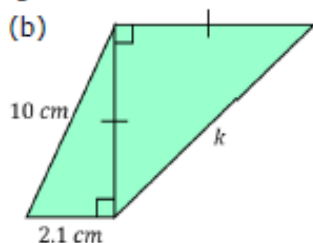


Find the missing lengths.

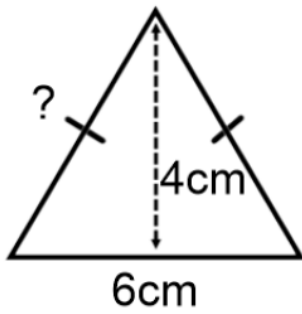
(a)



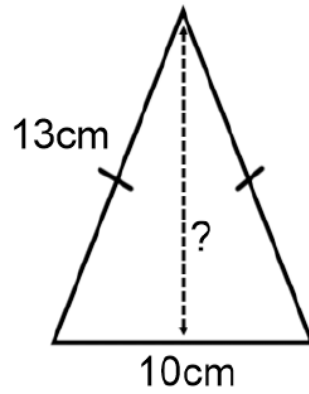
(b)



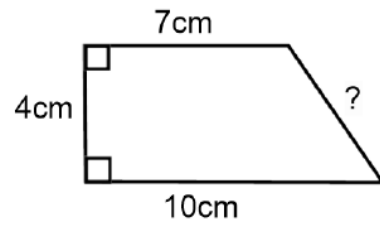
g)



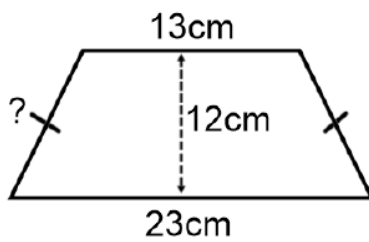
h)



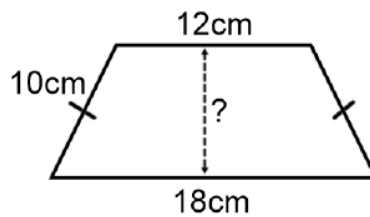
i)



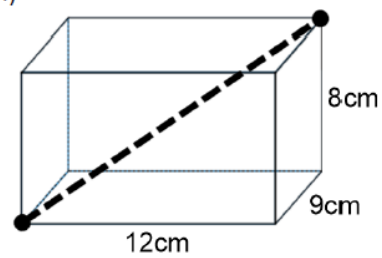
j)



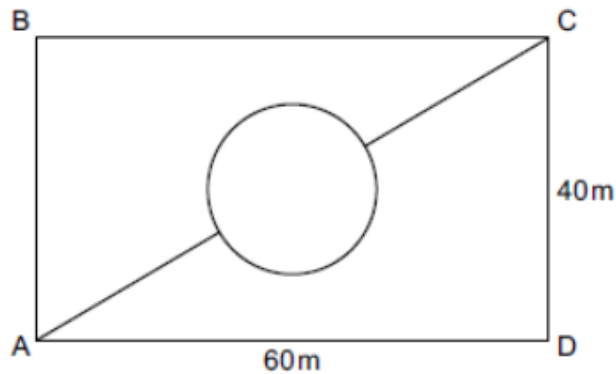
k)



l)



The rectangle ABCD represents a park.



Not to scale

The lines show all the paths in the park.

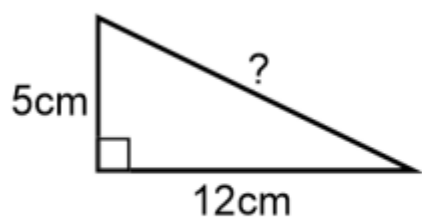
The circular path is in the centre of the rectangle and has a diameter of 10m.

Calculate the shortest distance from A to C across the park, using only the paths shown.

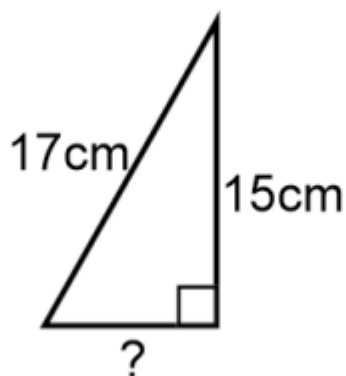
C2. Missing angles

Do now: find the missing lengths

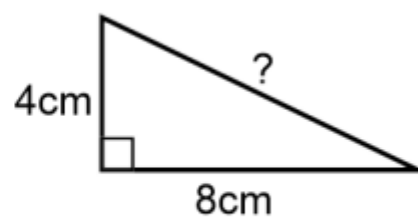
a)



b)



c)



Worked Example

$$\sin(x) = \frac{1}{2}$$

Your Turn

$$\sin(x) = \frac{2}{5}$$

Task

Find 'x'. Give your solution to 2 decimal places.

1. $\sin(x) = 0$

7. $\cos(x) = 0$

2. $\sin(x) = \frac{1}{5}$

8. $\cos(x) = \frac{1}{5}$

3. $\sin(x) = \frac{2}{5}$

9. $\cos(x) = \frac{2}{5}$

4. $\sin(x) = \frac{3}{5}$

10. $\cos(x) = \frac{3}{5}$

5. $\sin(x) = \frac{4}{5}$

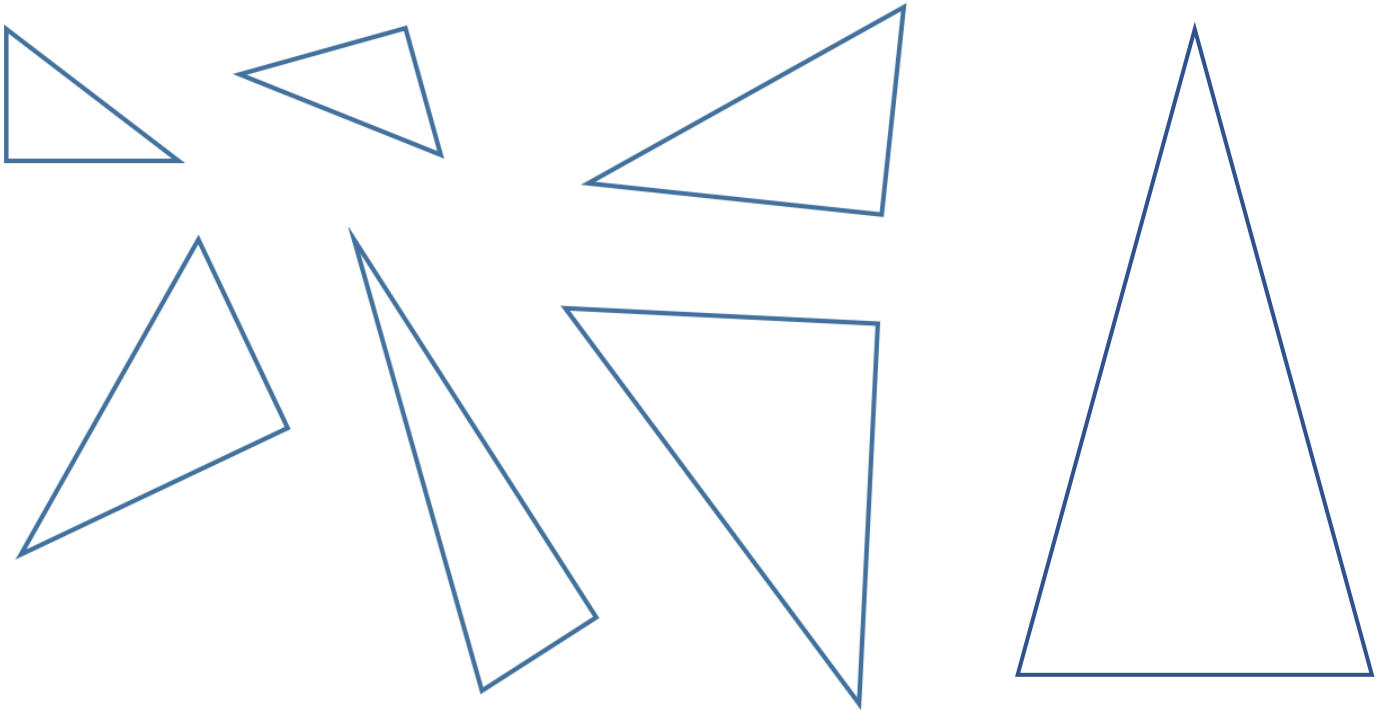
11. $\cos(x) = \frac{4}{5}$

6. $\sin(x) = 1$

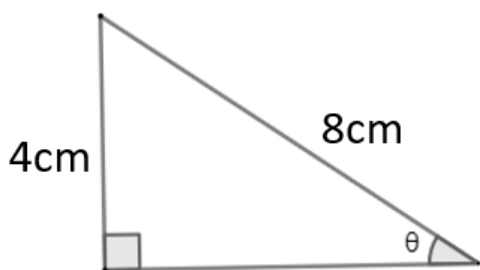
12. $\cos(x) = 1$

Operation	Inverse operation

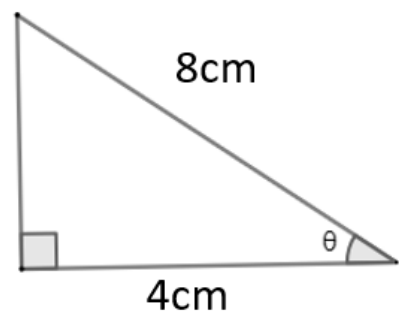
Labelling triangles

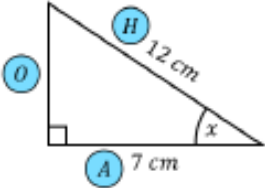
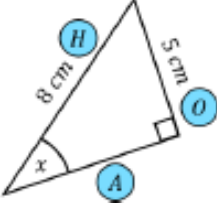
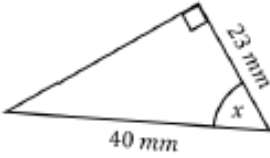
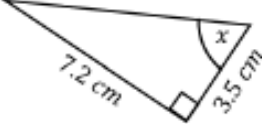
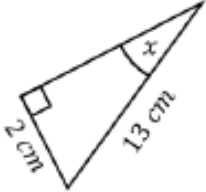

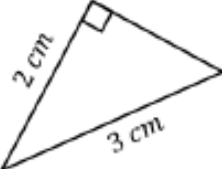


Worked Example

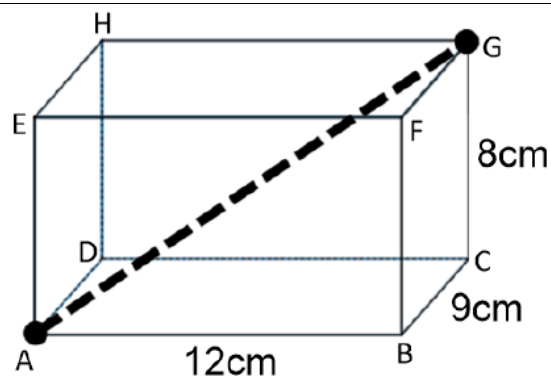
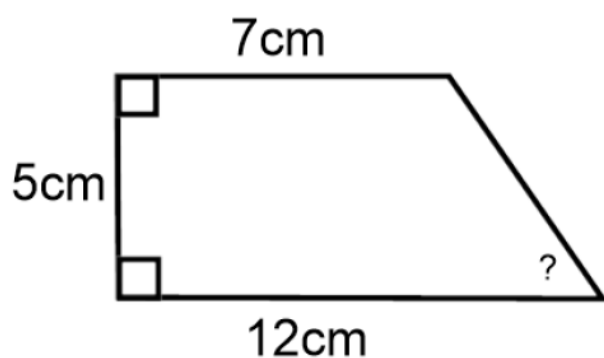


Your Turn



Labelled diagram	Choose ratio	Substitute into formula	Rearrange formula	Answer (1dp)
	cos	$\cos x = \frac{7}{12}$	$x = \cos^{-1}\left(\frac{7}{12}\right)$	
	sin			
				
				
				
				
		$\cos x = \frac{2}{3}$		
			$x = \tan^{-1}\left(\frac{15}{11}\right)$	

Extensions



Find the size of the angle between the line AG and the plane ABCD.

The diagram shows a circle with centre O . Points A , B , C and D all lie on the circumference of the circle.

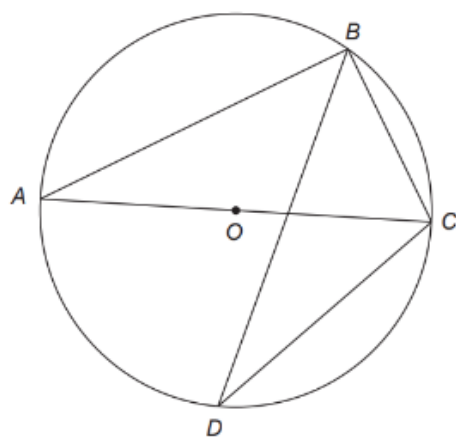


Diagram not drawn to scale

The radius of the circle is 3.6 cm, $BC = 4.1$ cm and $\widehat{BCD} = 93^\circ$.

Find the size of \widehat{DBC} , correct to 3 significant figures.

C3. Missing sides

Do now: solve the equations

$$(i) \frac{x}{3} = 5 \quad \bigg| \quad (ii) \frac{3}{x} = 5 \quad \bigg| \quad (iii) \frac{3}{x} = -5$$

Worked Example

$$\sin(30) = \frac{x}{5}$$

Your Turn

$$\cos(45) = \frac{x}{4}$$

Find 'x'. Give your solution to 2 decimal places.

1. $\tan(30) = \frac{x}{2}$

2. $\tan(45) = \frac{x}{2}$

3. $\sin(45) = \frac{x}{2}$

4. $\sin(45) = \frac{x}{4}$

5. $\frac{x}{4} = \sin(45)$

6. $x \times \sin(45) = 4$

7. $x \times \sin(30) = 4$

8. $x \times \cos(30) = 4$

9. $x \times \cos(30) = 8$

10. $x \times \cos(31) = 8$

$$\sin(15) = \frac{5}{x}$$

$$\cos(45) = \frac{5}{x}$$

Find 'x'. Give your solution to 2 decimal places.

1. $\cos(30) = \frac{2}{x}$

2. $\cos(45) = \frac{2}{x}$

3. $\sin(45) = \frac{2}{x}$

4. $\sin(45) = \frac{4}{x}$

5. $\sin(45) = \frac{8}{x}$

6. $\tan(45) = \frac{8}{x}$

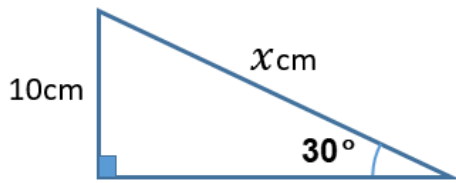
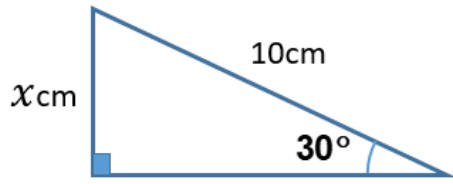
7. $\tan(45) = \frac{x}{8}$

8. $\cos(45) = \frac{x}{8}$

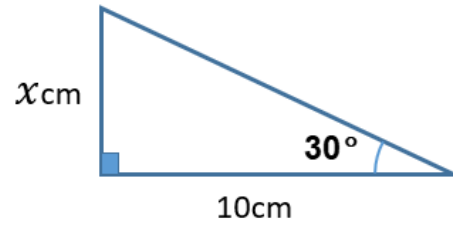
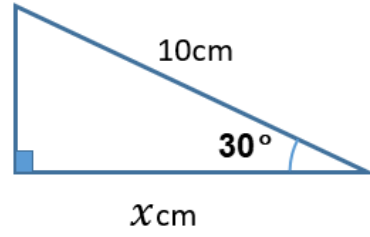
9. $\cos(45) = \frac{8}{x}$

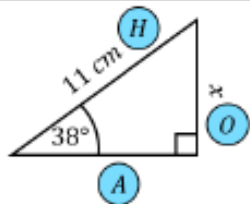
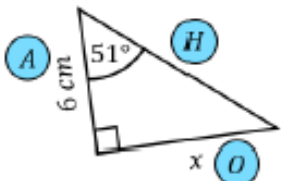
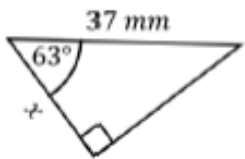
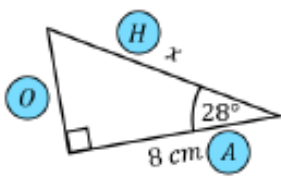
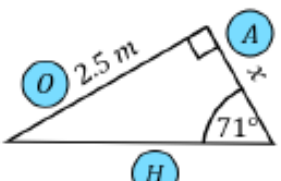
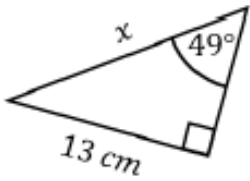
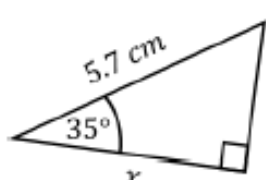
10. $\frac{8}{x} = \cos(45)$

Worked Example

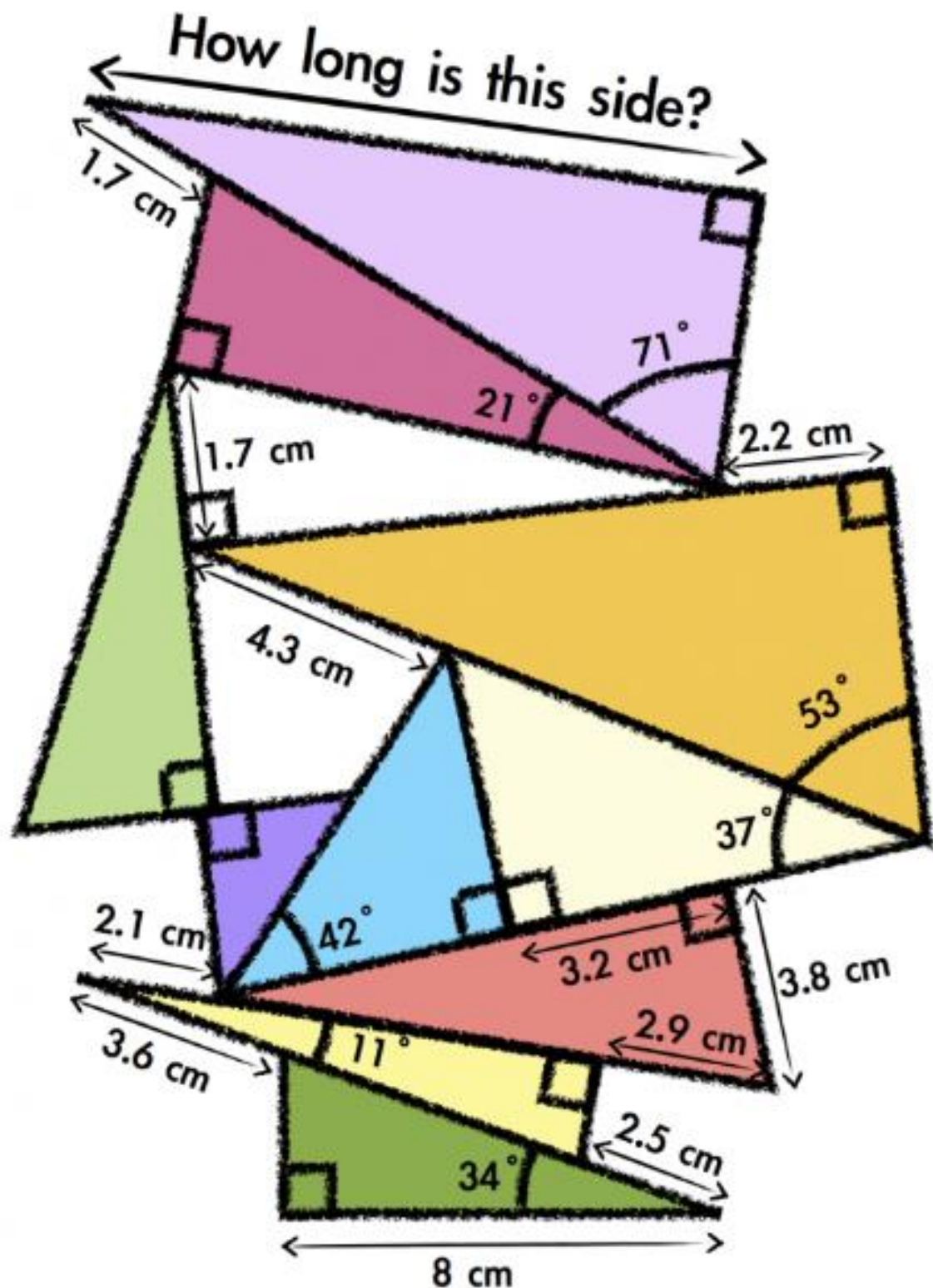


Your Turn



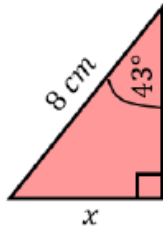
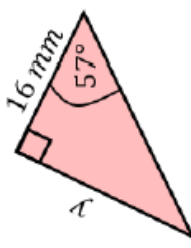
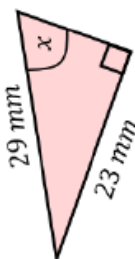
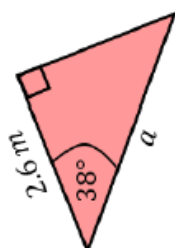
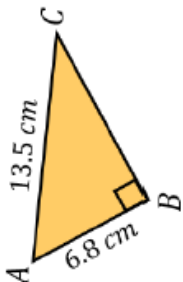

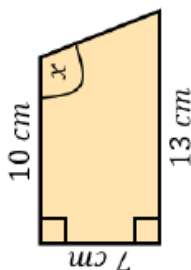
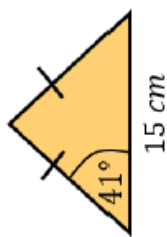
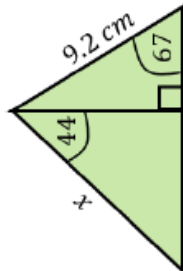
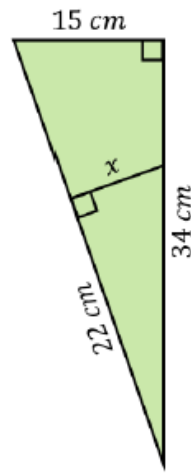
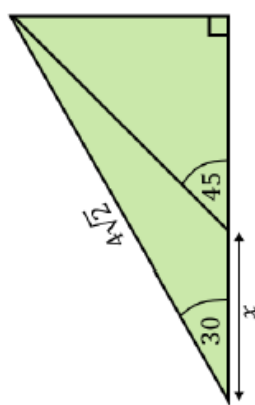
Labelled diagram	Choose ratio	Substitute into formula	Rearrange formula	Answer (1dp)
	sin	$\sin 38 = \frac{x}{11}$	$x = 11 \times \sin 38$	
	tan			
				
	cos	$\cos 28 = \frac{8}{x}$	$x = \frac{8}{\cos 28}$	
	tan			
				
				
		$\tan 68 = \frac{7}{x}$		

Trigonometry Pile Up!



Use this page for workings

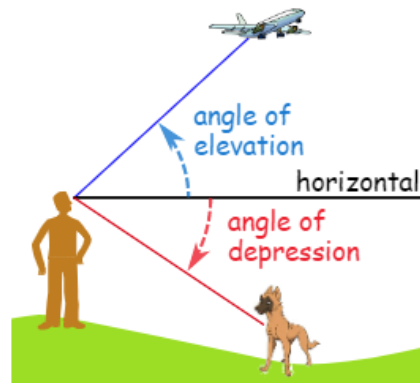
C4. Mixed practice

(a)	(b)	(c)	(d)
Find the value of x , to 1 decimal place. 	Find the value of y to 3 significant figures. 	Find angle x , to 1 decimal place. 	Find the value of a , to 1 decimal place. 
(e)	(f)	(g)	(h)
Find angle ACB to 3 significant figures. 	Find the perimeter of the triangle to 1 decimal place. 	Find angle x to 3 significant figures. 	Find the area of the isosceles triangle to 1 decimal place. 
(i)	(j)	(k)	
Find the length x to 3 significant figures. 	Find the value of x , giving your answer to 3 significant figures. 	Find the exact value of x . 	

C5a. Angles of elevation and depression

The "upwards" angle from the horizontal to a line of sight from the observer to some point of interest.

If the angle goes "downwards" it is called an Angle of Depression.



(a) Mercy stands 50 m away from the foot of a tower. When she looks to the top of the tower, the angle of elevation is 41° . Find the height of the tower.

(b) Rob is a passenger on a boat. The boat is 450 m from the foot of a cliff, which is 110 m high. Find the angle of elevation of the top of the cliff from the boat.

(c) Talha is on plane, looking down at the airport with an angle of depression of 36° . The height of the plane from the ground is 5 km . Find the distance from the plane to the airport.

(d) A pigeon flies down to the ground from the top of a tree at an angle of depression of 28° . The distance the pigeon flies is 15.8 m . How tall is the tree?

(e) Salisbury cathedral spire is 123 m tall. Guy stands 46 m from the cathedral spire. What is the angle of elevation of the top of the spire from where Guy is standing?

(f) A plane passes overhead at a height of 8000 m . A short time later, it is at an angle of elevation of 71° . How far away is the plane from its original position?

(g) A prison officer watches prisoners from a guard tower which is 10.5 m tall. He looks due North and can see two prisoners. The angle of depression of each of the prisoners is 18° and 23° . How far apart along the ground are the two prisoners?

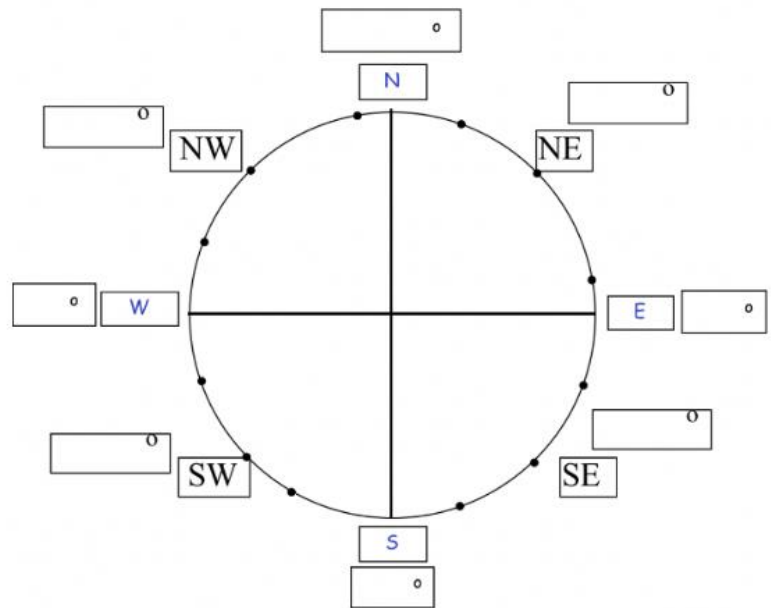
C5b. Bearings

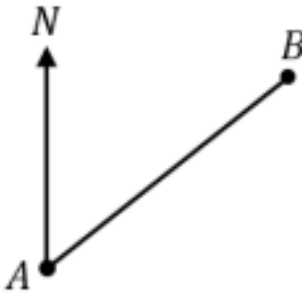

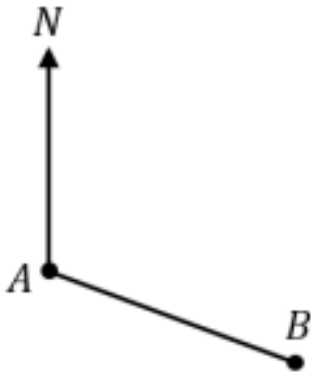

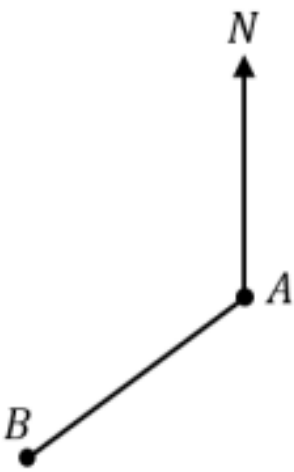
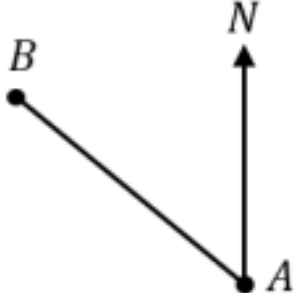
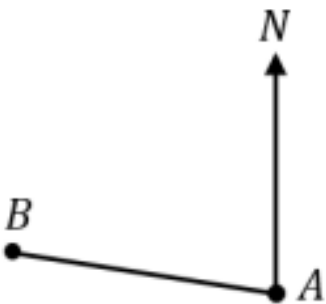


The 3 Rules of Bearings:

1.

2.

3.



(a)	(b)	(c)
		
(d)	(e)	(f)
		
(g)	(h)	(i)
		

Drawing Bearings

For each question, draw a line on the bearing given:

(a)

085°



(b)

040°



(c)

115°



(d)

155°



(e)

200°



(f)

310°



(g)

262°



(h)

148°



(i)

327°



Worked example

You try

<p>A hotel is located 3km east and 2km north of the beach. Calculate the bearing of the hotel from the beach to the nearest degree.</p>	<p>A ship sets off from port. It sails 3 miles due south and then 5 miles east. On what bearing is the port from the ship to the nearest degree?</p>
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TASK	
<p>A scout troop are hiking in a forest. Starting from their base, they walk 4.2km south followed by 7.1km west. They want to walk the shortest distance back to their base. On what bearing should the scouts walk?</p>	<p>A dragon has been wreaking havoc in a local village. It is then chased away by a knight. The dragon flies 3 miles due south followed by 4 miles due west.</p> <ol style="list-style-type: none"> Work out the distance between the dragon and the village. Find the bearing of the dragon from the village to the nearest degree.
<p>The diagram shows the positions of a tower and a tree. The tree is 2.1km South of the tower and 4.5km East of the tower.</p> <ol style="list-style-type: none"> Work out the distance between the tower and the tree to one decimal place. Work out the bearing of the tree from the tower to the nearest degree. 	<p>A aeroplane flies 22 miles due south followed by 34 miles due east. On what bearing is the aeroplane's starting position from its current position to the nearest degree?</p>

<p>A buoy is located 214m west and 185m south of a ship. Calculate the bearing of the buoy from the ship to the nearest degree.</p>	<p>Two ships sail away from port. Ship A sails 6.4miles due south followed by 12.4miles due west. Ship B sails 3miles due south followed by 10.1 miles due east.</p> <ol style="list-style-type: none"> Work out the distance between the two ships to one decimal place. Work out the bearing of Ship B from Ship A to the nearest degree.
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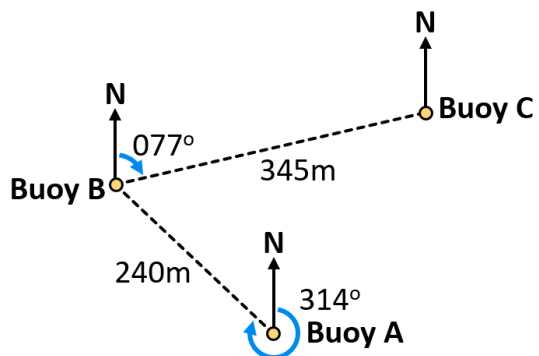
Examples	
<p>An aeroplane flies 50 miles on a bearing of 112°. How far due East has it travelled to the nearest one decimal place?</p>	<p>Ground troops are on a bearing of 072° from their base. They know that they are a distance of 6km due East of the base but don't know their distance due North. Work out the shortest distance between the base and the ground troops to the nearest one decimal place.</p>

Task	
<p>The following diagram represents the positions and bearings of two helicopters. Helicopter B is 1.2km away from helicopter A on a bearing of 122°. How far north is helicopter A from helicopter B?</p>	<p>Animal conservationists are tracking the position of a polar bear. The polar bear's position is 3km in an eastward direction and is on a bearing of 104°. What is the shortest distance between the polar bear and the conservationists.</p>

Extensions

The following diagram represents the positions and bearings of three buoys floating in the ocean.

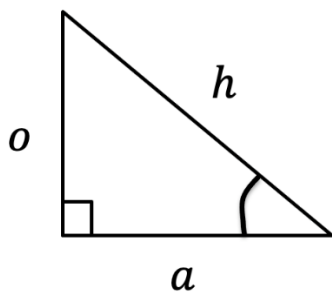
- [a] How far west is Buoy B from Buoy C.
- [b] How far east is Buoy A from Buoy B.
- [c] How far north is Buoy C from Buoy A.



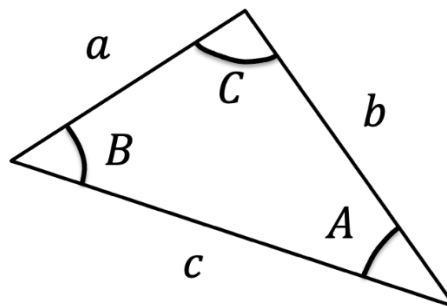
A ship sails on a bearing of 074° for 10 miles followed by a bearing of 131° for 15 miles. Work out the bearing of the ship from its starting position to the nearest degree.

C6. The Sine Rule

Right-Angled Triangles:

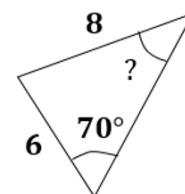


Non-Right-Angled Triangles:



We label the sides a, b, c and their corresponding OPPOSITE angles A, B, C

You have	You want	Use
#1: Two angle-side opposite pairs	Missing angle or side in one pair	Sine rule

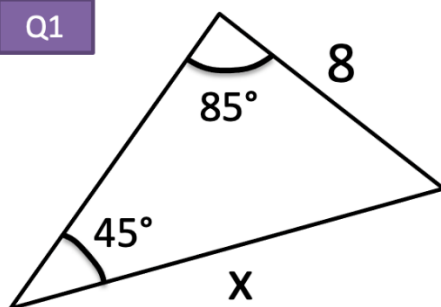


Sine Rule:

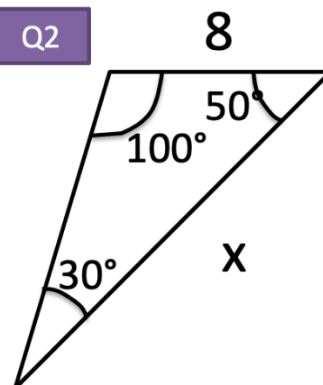
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Examples

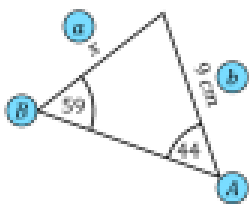
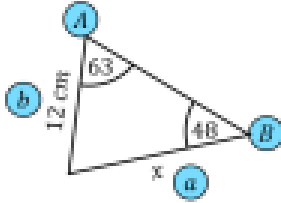
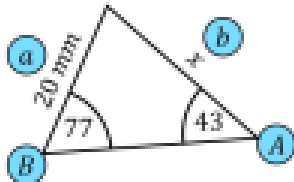
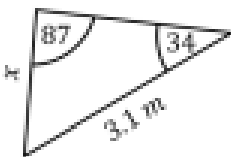
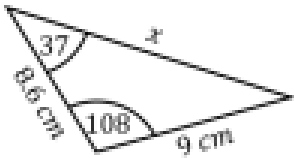
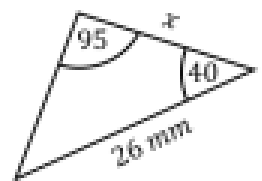
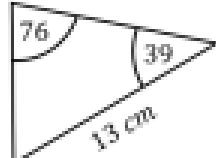
Q1



Q2

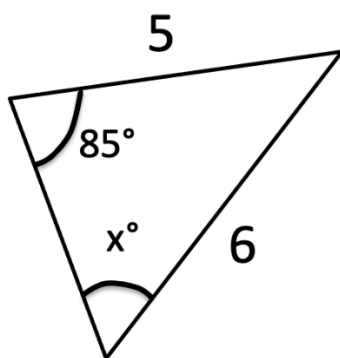


Examples 2

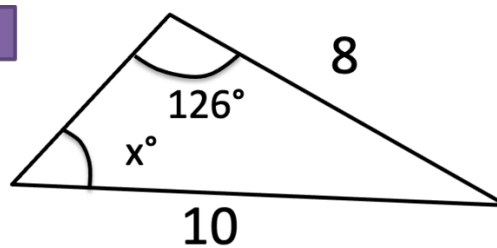
Labelled diagram	Substitute into formula	Rearrange formula	Length (1dp)
	$\frac{x}{\sin 44} = \frac{9}{\sin 59}$	$x = \frac{9 \times \sin 44}{\sin 59}$	
	$\frac{x}{\sin 63} = \frac{12}{\sin 48}$		
			
			
			
			
	$\frac{x}{\sin 65} = \frac{13}{\sin 76}$		
		$x = \frac{3.5 \times \sin 36}{\sin 68}$	

Examples 2

Q3

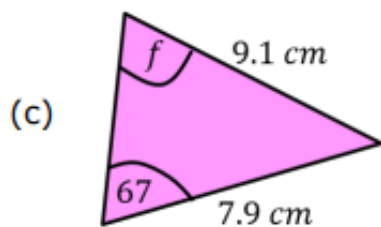
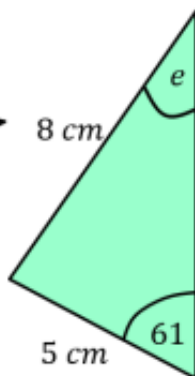
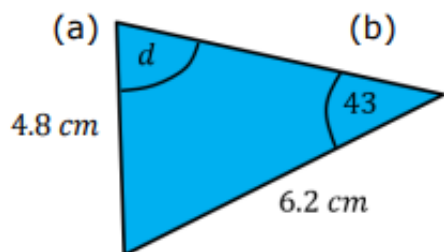


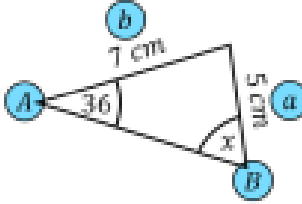
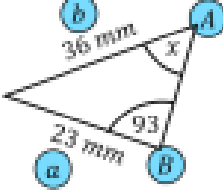
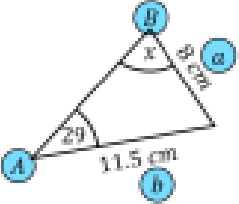
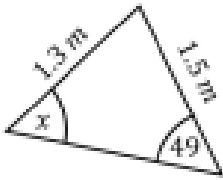
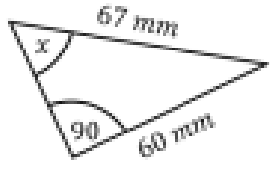
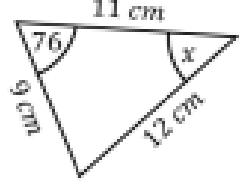
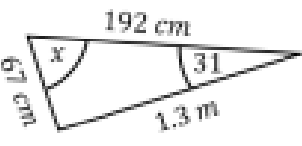
Q4



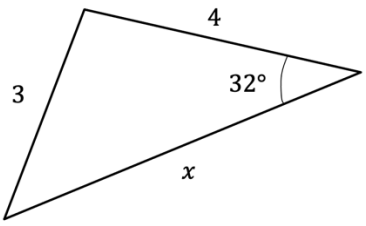
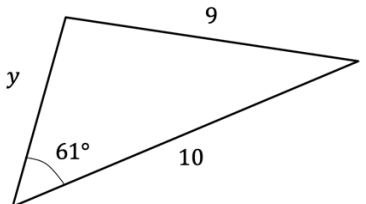
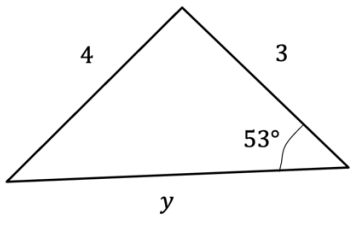
Test your understanding

Find the missing angle.

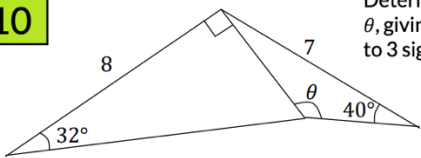
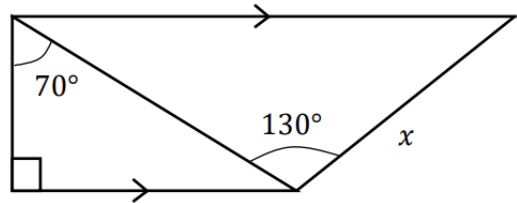


Labelled diagram	Substitute into formula	Rearrange formula	Acute Angle (1dp)
	$\frac{\sin 36}{5} = \frac{\sin x}{7}$	$\sin x = \frac{7 \times \sin 36}{5}$	$x = 55.4^\circ$
	$\frac{\sin x}{23} = \frac{\sin 93}{36}$		
			
			
			
			
			
		$\sin x = \frac{5 \times \sin 47}{10}$	

Extension

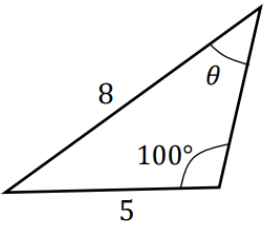
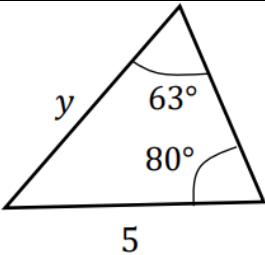
 <p>A triangle with a left side of length 3, a top side of length 4, and a bottom side of length x. The angle between the top and bottom sides is 32°.</p>	 <p>A triangle with a left side of length y, a top side of length 9, and a bottom side of length 10. The angle between the left and bottom sides is 61°.</p>	 <p>A triangle with a left side of length 4, a right side of length 3, and a bottom side of length y. The angle between the right and bottom sides is 53°.</p>
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Problem Solving

<div style="display: flex; align-items: center;"> <div style="background-color: #90EE90; padding: 2px 5px; margin-right: 10px; font-weight: bold;">10</div> <div style="text-align: center;">  <p>A triangle with a left side of length 8 and a right side of length 7. The bottom-left angle is 32° and the bottom-right angle is 40°. The top angle is labeled θ. A right-angle symbol is shown at the top vertex.</p> </div> <div style="margin-left: 10px;"> <p>Determine the angle θ, giving your answer to 3 significant figures.</p> </div> </div>	<div style="text-align: center;">  <p>A triangle with a left side of length 8. The top-left angle is 70° and the bottom-right angle is 130°. The right side is labeled x. A right-angle symbol is shown at the bottom-left vertex. Arrows on the top and bottom horizontal sides indicate they are parallel.</p> </div> <p>Determine the length of x, giving your length correct to 3 significant figures.</p>
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C7. The Cosine Rule

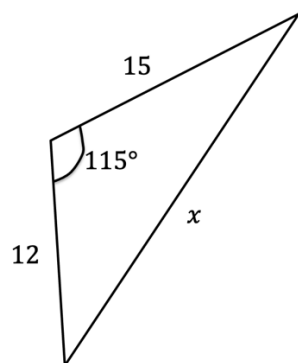
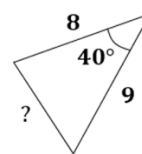
Do now: work out the unknown, giving your answers to 3 significant figures

<p>1</p> 	<p>2</p> 
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#2 Two sides known
and a missing side
opposite a known
angle

Remaining side

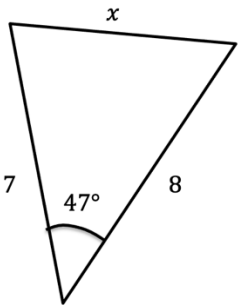
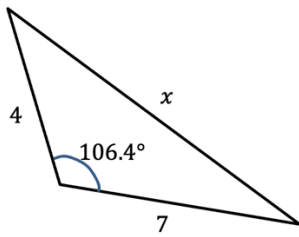
Cosine rule

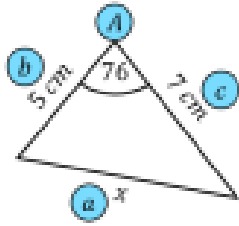
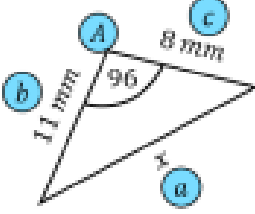
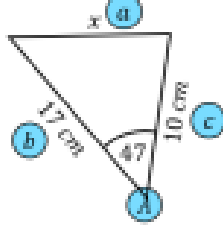
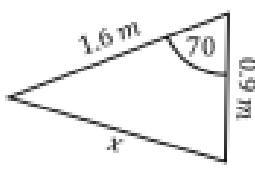
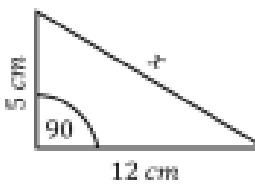
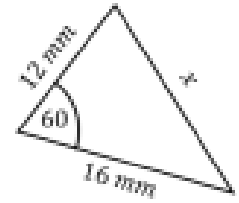


Cosine Rule:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Examples

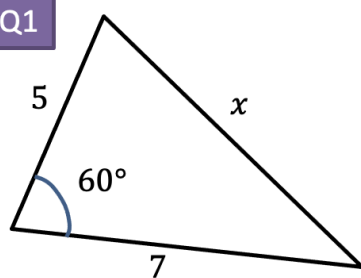
	
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Labelled diagram	Substitute into formula	x^2	x to 1dp
 <p>A triangle with vertices A, B, and C. Side AB is 5 cm, side AC is 7 cm, and the angle at vertex A is 76 degrees. The side opposite vertex A is BC, labeled x.</p>	$x^2 = 7^2 + 5^2 - 2 \times 7 \times 5 \times \cos 76$	$x^2 = 57.065..$	
 <p>A triangle with vertices A, B, and C. Side AB is 11 mm, side AC is 8 mm, and the angle at vertex A is 96 degrees. The side opposite vertex A is BC, labeled x.</p>	$x^2 = 11^2 + 8^2 - 2 \times 11 \times 8 \times \cos 96$		
 <p>A triangle with vertices A, B, and C. Side AB is 17 cm, side AC is 10 cm, and the angle at vertex A is 47 degrees. The side opposite vertex A is BC, labeled x.</p>			
 <p>A triangle with vertices A, B, and C. Side AB is 1.6 m, side AC is 0.6 m, and the angle at vertex A is 70 degrees. The side opposite vertex A is BC, labeled x.</p>			
 <p>A right-angled triangle with vertices A, B, and C. Side AB is 5 cm, side AC is 12 cm, and the angle at vertex A is 90 degrees. The side opposite vertex A is BC, labeled x.</p>			
 <p>A triangle with vertices A, B, and C. Side AB is 12 mm, side AC is 16 mm, and the angle at vertex A is 60 degrees. The side opposite vertex A is BC, labeled x.</p>			
	$x^2 = 32^2 + 14^2 - 2 \times 32 \times 14 \times \cos 53$		

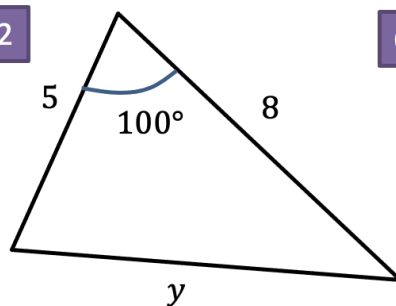
Extensions

Use the cosine rule to determine the missing angle/side.

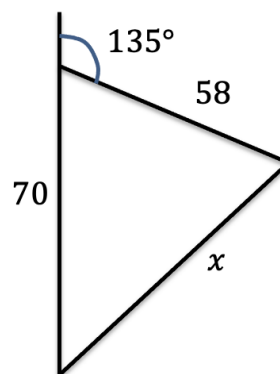
Q1



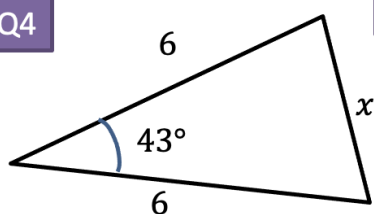
Q2



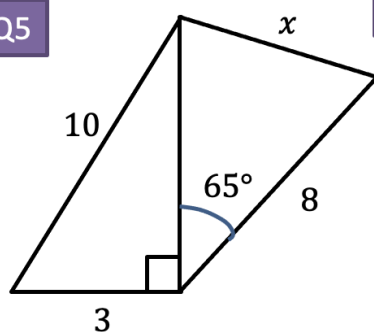
Q3



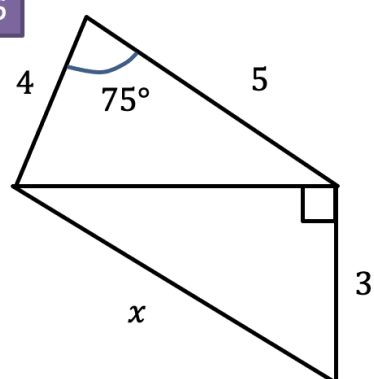
Q4



Q5

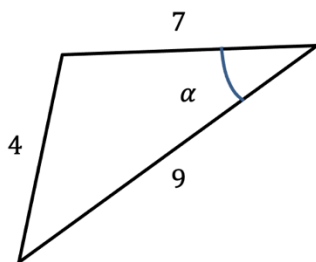


Q6

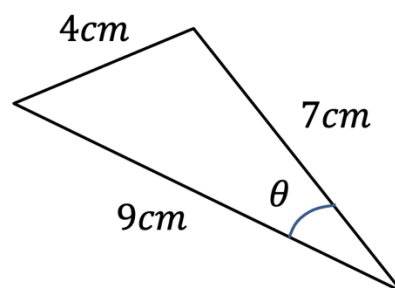
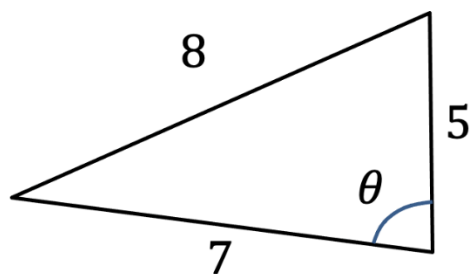


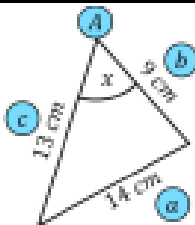
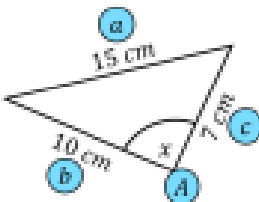
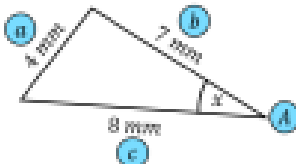
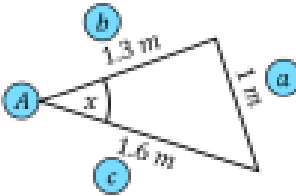

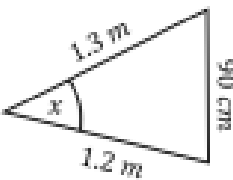
You have	You want	Use
All three sides	An angle	Cosine rule

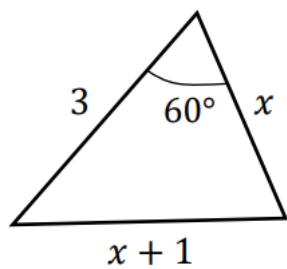
$$a^2 = b^2 + c^2 - 2bc \cos A$$



Test your understanding

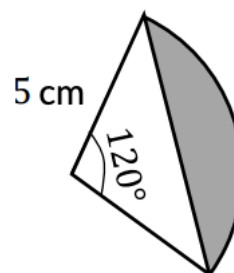


Labelled diagram	Substitute into formula	Rearrange formula	Angle (1dp)
	$14^2 = 9^2 + 13^2 - 2 \times 9 \times 13 \times \cos x$	$\cos x = \frac{9^2 + 13^2 - 14^2}{2 \times 9 \times 13}$	$x = 76.7^\circ$
	$15^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \times \cos x$	$\cos x = \frac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$	
	$4^2 = 7^2 + 8^2 - 2 \times 7 \times 8 \times \cos x$		
			
			
			
		$\cos x = \frac{6^2 + 5^2 - 3^2}{2 \times 6 \times 5}$	



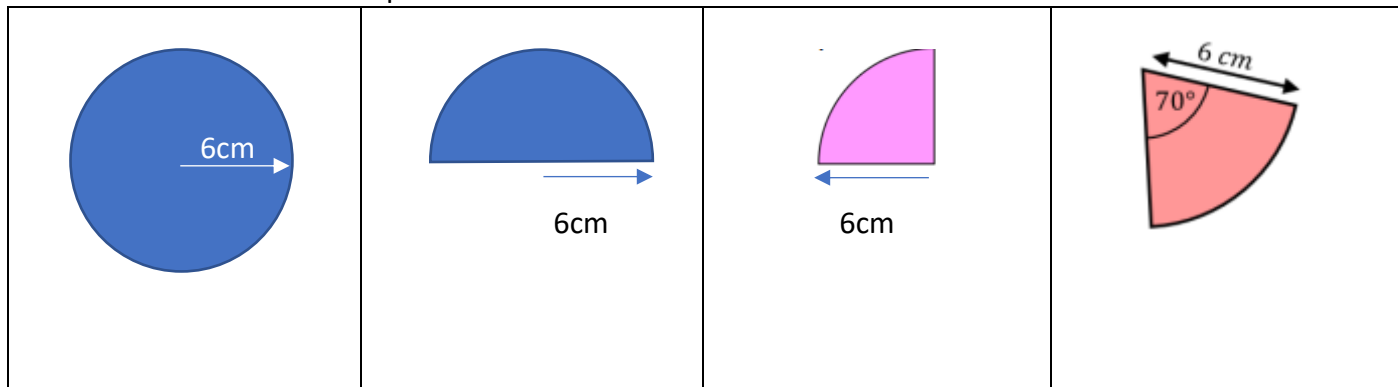
Determine the value of x .

The diagram shows a third-circle. Determine the perimeter of the shaded region.



C8. Area of non-right angled triangles

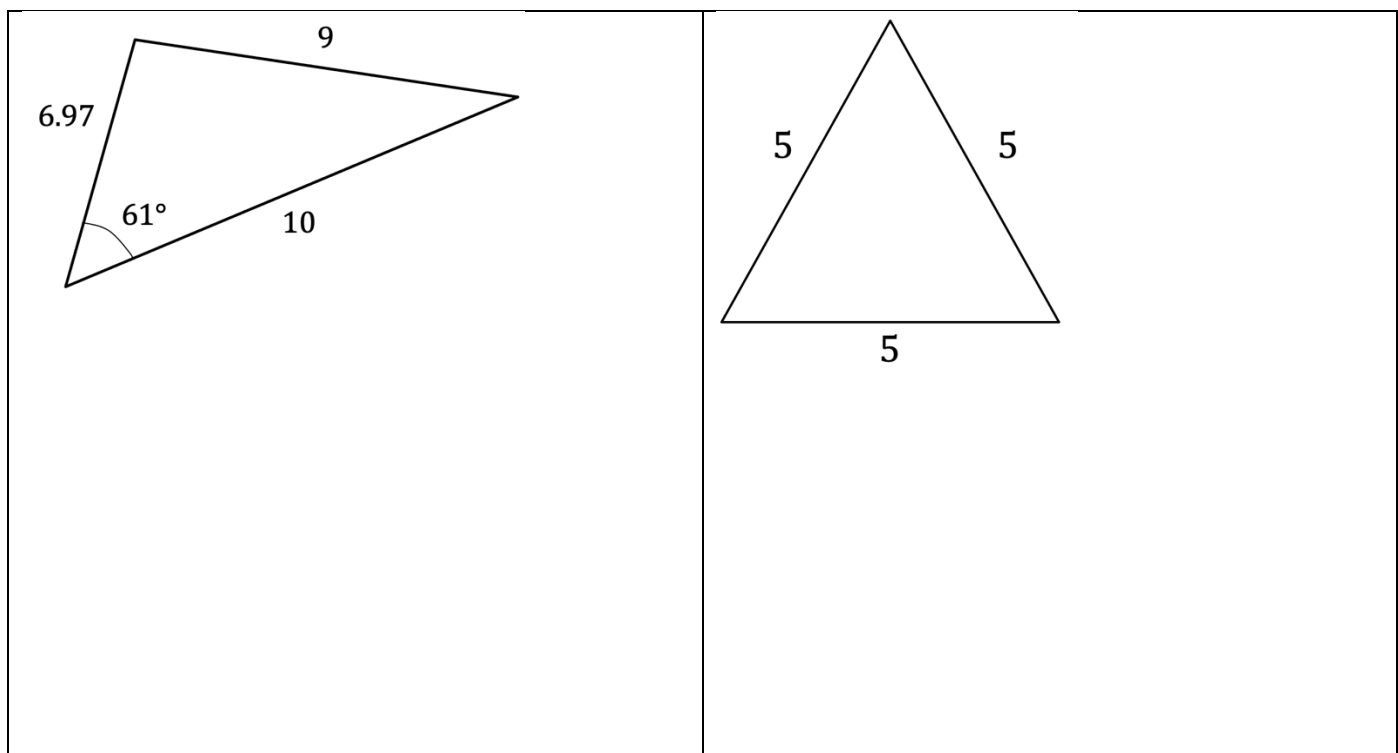
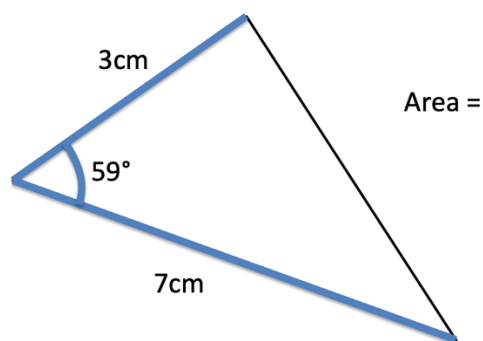
Do now: Find the area of the parts of circles



Examples

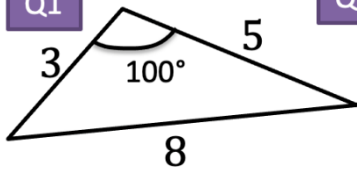
$$\text{Area} = \frac{1}{2} a b \sin(C)$$

Where C is the angle wedged between two sides a and b.

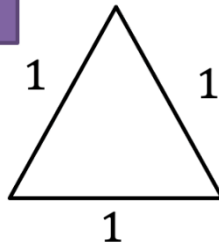


Calculate the areas of the triangles

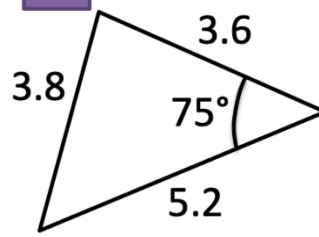
Q1



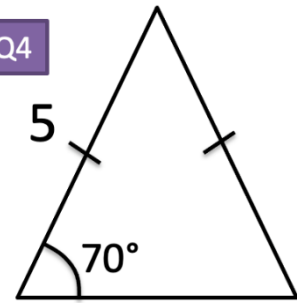
Q2



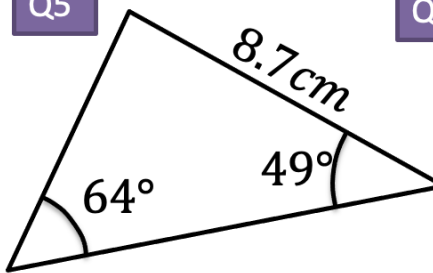
Q3



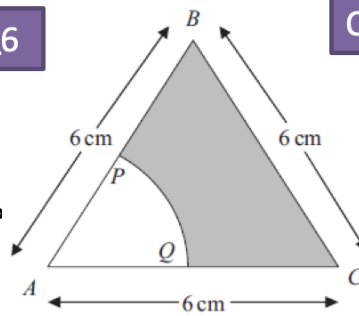
Q4



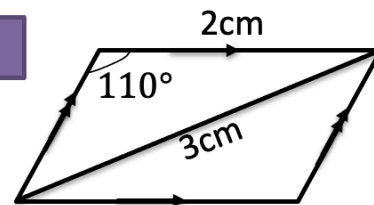
Q5



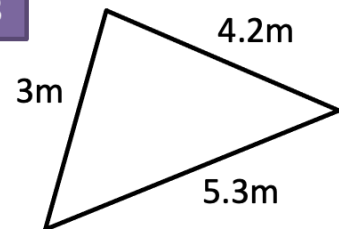
Q6



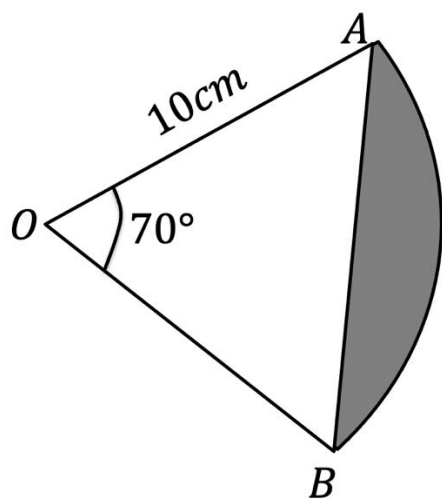
Q7



Q8



P is the midpoint of AB and Q the midpoint of AC . APQ is a sector of a circle. Find the shaded area.

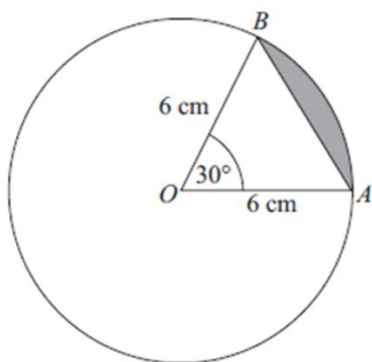


OAB is a sector of a circle, centred at O .
Determine the area of the shaded segment.

Area of sector =

Area of triangle =

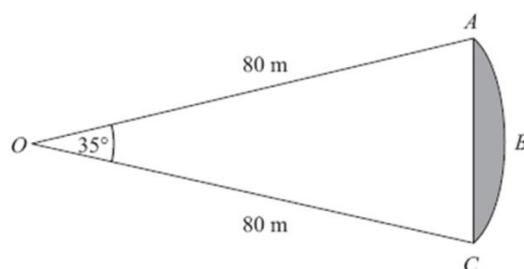
Area of segment =
=



The diagram shows a circle, centre O .
 A and B are points on the circle.
 $OA = OB = 6$ cm.

The value of $\sin 30^\circ = \frac{1}{2}$

Work out the area of the shaded segment.
Give your answer in terms of π .

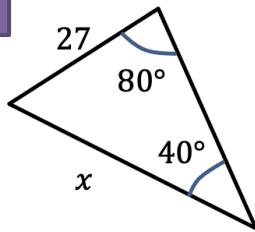


ABC is an arc of a circle centre O with radius 80 m.
 AC is a chord of the circle.
Angle $AOC = 35^\circ$.

Calculate the area of the shaded region.
Give your answer correct to 3 significant figures.

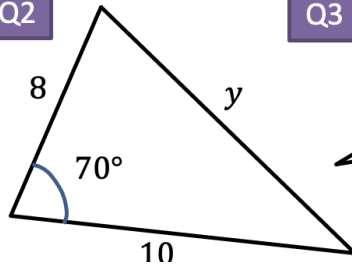
C9. Mixed problems

Q1



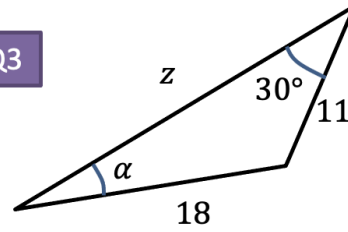
a) $x =$
b) Area =

Q2

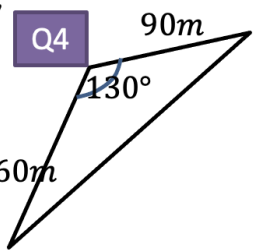


$y =$
Area =

Q3

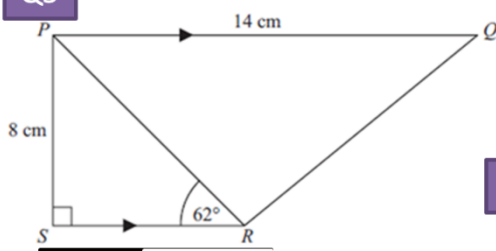


$\alpha =$
 $z =$
Area =

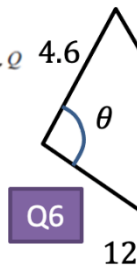


Perimeter =

Q5



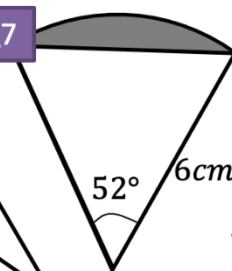
$QR =$



Q6

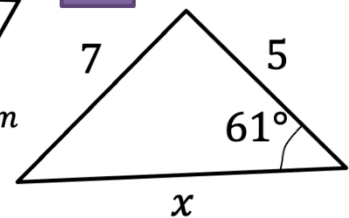
$\theta =$

Q7



Area =

Q8



$x =$
Area =